Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
Confederaziun svizra

Federal Communications Commission ComCom
Federal Office of Communications

Calculation of Base and Additional Prices

Calculation of base prices

The process described below establishes a set of base prices in compliance with Rule 3.3.32 of the Auction Rules. The process considers the maximum amount by which the winning bid(s) could be reduced (the maximum bid discount(s)) while maintaining the original combination of winning bids as a winning combination. Bid discounts can also be interpreted as surplus enjoyed by the bidder relative to the bids submitted.

Notation

Let $N = \{1...n\}$ be the set of bidders participating in the auction, and B_j the set of all bids submitted by bidder $j \in N$. The set of all bids is then given by $B = B_1 \cup B_2 ... \cup B_n$.

Let b_i denote the bid amount specified in bid $i \in B$.

Let W be the set of winning bidders determined pursuant to Rules 3.3.25 - 3.3.29 of the Auction Rules when using the bids in B, and ω the corresponding set of winning bids, with ω_j denoting the winning bid selected from winner $j \in W$, and b_{ω_j} the bid amount specified in this winning bid.

Let $v(\omega) = \sum_{i \in \omega} b_i$ be the sum of winning bid amounts over all bids in ω.

Let W^{-C}, ω^{-C} and $v\left(\omega^{-C}\right)$ respectively denote the set of winners, the set of winning bids, and the sum of winning bids that would result from the application of Rules 3.3.25 - 3.3.29 if all bids made by bidders in $C \subseteq N$ were discarded.

$\sigma(C) = \left(v(\omega) - v\left(\omega^{-C}\right)\right)$ then measures the reduction in the sum of winning bid amounts if all bids submitted by bidders in C were discarded.

Process

The process for determining base prices associated with winning bids is as follows:

a) For each winning bidder $j \in W$ calculate $\sigma(j) = \left(v(\omega) - v\left(\omega^{-j}\right)\right)$. This is the maximum amount by which the winning bid of bidder j could be reduced without changing the outcome of the winner determination process (ignoring that bidders must pay at least the reserve prices associated with the lots they win). If the bids of bidder j were to be reduced by more than $\sigma(j)$, that bidder would no longer form part of the group of winning bidders because some other combination of bids would have a greater value. $\sigma(j)$ also denotes the maximum surplus that could be achieved by bidder j.

b) Because winners need to pay at least the reserve price associated with the lots they are winning, the maximum discount that could be given to bidder j will be smaller whenever $\rho_j = b_{\omega_j} - m_j < \sigma(j)$, where m_j is the sum of reserve prices of the lots in the winning bid. ρ_j denotes the maximum discount to reserve that is available to bidder j.

Therefore, the maximum discount that can be given to winner $j \in W$ is given by:

$$d_j^{\max} = \min\left[\ \rho_j, \quad \sigma(j)\ \right]$$

c) Generate a list of constraints C for a linear program and initialise it as $C = \left\{\ \forall j \in W:\ d_j \geq 0, d_j \leq d_j^{\max}; \quad \sum_{j \in W} d_j \leq \sigma(W)\ \right\}$.

d) Pursuant to the third condition of Rule 3.3.32, minimise the sum of base prices paid by all bidders, i.e. maximise the sum of discounts over all winning bidders: $\max \sum_{j\ \in\ W} d_j$, subject to the constraints in C. There may be multiple solutions to this maximisation problem. If this is the case, pick a random solution $d^\star$.

e) For all winners $j \in W$ reduce the bid amounts of all bids submitted by Bidder j by $d_j^\star$; if this yields a negative bid amount, set this bid amount to zero. For all other bidders $j \notin W$, we leave their bid amounts unchanged. Therefore, we define modified bids $\tilde{b_i} = \max\left[b_i - d_j^*, 0\right]$ for $i \in B_j, j \in W$, $\tilde{b_i} = b_i$ for $i \in B_j, j \notin W$, which together define a modified bid set $\tilde{B}$.

f) Solve the winner determination problem for $\tilde{B}$. This yields $\tilde{W}$, $\tilde{\omega}$ and $\tilde{v}(\tilde{\omega})$.

g) Calculate the sum of bid amounts of the original winning bids using the reduced bid amounts, $\tilde{v}(\omega) = \sum_{i \in \omega} \tilde{b}_i$. If $\tilde{v}(\omega) < \tilde{v}(\tilde{\omega})$, then winners cannot be given the discounts $d^\star$ calculated in Step d), as there exists a different combination of bidders $\tilde{W}$ that could provide greater revenues if bids were modified by these discounts. A new constraint is required; proceed to Step h).

Otherwise, the maximum discount that can be granted to winning bidders is equal to $\sum_{j \in W} d_j^\star = D^\star$. Go to Step i).

h) If any of the original winners is no longer among the new winning bidders $\tilde{W}$ calculated in Step f), then add the following constraint to the list of constraints C:

$$\sum_{j \in L} d_j \leq \sigma(L)$$

where L denotes the set of winners in the original solution W that are no longer among the new winners $\tilde{W}$ calculated in Step f). Go to Step d).

i) If there was a single solution to the optimisation problem in Step d) these discounts can be used to calculate the base prices. Proceed to Step n).

Otherwise, pursuant to the fourth condition in Rule 3.3.32, we will need to find a combination of discounts that distributes the total available surplus (i.e. the total available discount) evenly across bidders relative to the maximum surplus calculated for each winner. This is done by minimising the sum of squares of the difference between the maximum surplus and the actual surplus (equal to the bid discount) applied in calculating prices for each bidder: $\min \sum_{j \in W} \left(d_j - \sigma(j)\right)^2$, subject to the constraints in C and the constraint that the sum of the individual discounts by winners must be equal to the maximum total discount that can be granted to winning bidders ($D^\star$) as calculated in Step g). Let $d^{\star\star}$ be the solution to this minimisation problem.

j) For all bidders $j \in W$ reduce the bid amounts for all bids submitted by Bidder j by $d_j^{\star\star}$; if this yields a negative bid amount, set this bid amount to zero. For all other bidders $j \notin W$, we leave their bid amounts unchanged. Therefore, we the modified bids are defined as $\tilde{b_i} = \max\left[b_i - d_j^{\star\star}, 0\right]$ for $i \in B_j, j \in W$, $\tilde{b_i} = b_i$ for $i \in B_j, j \notin W$, giving together a set of modified bids $\tilde{B}$.

k) Solve the winner determination problem when using $\tilde{B}$. This yields $\tilde{W}$, $\tilde{\omega}$ and $\tilde{v}(\tilde{\omega})$.

l) Calculate the sum of bid amounts of the original winning bids using the reduced bid amounts, $\tilde{v}(\omega) = \sum_{i \in \omega} \tilde{b}_i$. If $\tilde{v}(\omega) < \tilde{v}(\tilde{\omega})$, then winners cannot be given the discounts $d_j^{\star\star}$ calculated in Step k), as there exists a different combination of bidders $\tilde{W}$ that could provide greater revenues if bids were modified by these discounts. A new constraint is required; proceed to Step m).

Otherwise, the discounts $d_j^{\star\star}$ calculated in step k) can be used to calculate the base prices. Proceed to Step n).

m) If any of the original winners is no longer among the new winning bidders calculated in Step k), then add the following constraint to the list of constraints C:

$$\sum_{j \in L} d_j \leq \sigma(L)$$

where L denotes the set of winners in the original solution W that are no longer among the new winners $\tilde{W}$ calculated in Step k). Go to Step i).

n) The base price for each winner is the bid amount of its winning bid reduced by the discount calculated in the previous step, i.e. $p_j = b_{\omega_j} - d_j^\star$ or $p_j = b_{\omega_j} - d_j^{\star\star}$ respectively for all $j \in W$.

Calculation of additional prices.

The process described below establishes a set of additional prices in compliance with Rule 4.8.2 of the Auction Rules. The process considers the maximum amount by which the winning assignment bid(s) could be reduced (the maximum bid discount(s)) while maintaining the original combination of winning assignment bids as a winning combination. Bid discounts can also be interpreted as surplus enjoyed by the bidder relative to the assignment bids submitted.

Notation

Let $N = \{1...n\}$ be the set of bidders participating in the assignment of a particular band, and B_j the set of all assignment bids submitted by bidder $j \in N$ for the assignment of this band. The set of all assignment bids for this band is then given by $B = B_1 \cup B_2 ... \cup B_n$.

Let b_i denote the bid amount specified in assignment bid $i \in B$.

Let ω be the set of winning assignment bids determined pursuant to Rules 4.7.2 - 4.7.4 of the Auction Rules when using the bids in B, with ω_j denoting the winning assignment bid selected from bidder $j \in N$, and b_{ω_j} the bid amount specified in this winning assignment bid.

Let $v(\omega) = \sum_{i \in \omega} b_i$ be the sum of winning bid amounts over all bids in ω.

Let ω^{-C} and $v(\omega^{-C})$ respectively denote the set of winning assignment bids, and the sum of winning assignment bids that would result from the application of Rules 4.7.2 - 4.7.4 if the bidders in $C \subseteq N$ had not submitted any positive assignment bids.

$\sigma(C) = \left(v(\omega) - v(\omega^{-C})\right)$ then measures the reduction in the sum of winning bid amounts if all bidders in C do not express any preference for particular assignments.

Process

The process for determining the additional prices associated with winning bids is as follows:

a) For each bidder $j \in N$ calculate $\sigma(j) = \left(v(\omega) - v(\omega^{-j})\right)$. This is the maximum amount by which the winning assignment bid of bidder j could be reduced without changing the outcome of the assignment winner determination process. $\sigma(j)$ denotes the maximum surplus that could be achieved by bidder j.

b) Given that there are no minimum prices for assignment bids, the maximum discount that can be given to bidder $j \in N$ is given by:

$$d_j^{\max} = \sigma(j).$$

c) Generate a list of constraints C for a linear program and initialise it as $C = \{ \ \forall j \in N: \ d_j \geq 0, d_j \leq d_j^{\max}; \quad \sum_{j \in N} d_j \leq \sigma(N) \ \}$.

d) Pursuant to the third condition of Rule 4.8.2, minimise the sum of additional prices paid by all bidders, i.e. maximise the sum of discounts over all bidders: $\max \sum_{j \in N} d_j$, subject to the constraints in C. There may be multiple solutions to this maximisation problem. If this is the case, pick a random solution $d^\star$.

e) For all bidders $j \in N$ reduce the bid amounts of all assignment bids submitted by Bidder j by $d_j^\star$; if this yields a negative bid amount, set this bid amount to zero. Therefore, we define modified assignment bids $\tilde{b}_i = \max\left[b_i - d_j^*, 0\right]$ for $i \in B_j, j \in N$ which define a modified assignment bid set $\tilde{B}$.

f) Solve the winner determination problem for $\tilde{B}$. This yields $\tilde{\omega}$ and $\tilde{v}(\tilde{\omega})$.

g) Calculate the sum of bid amounts of the original winning assignment bids using the reduced bid amounts, $\tilde{v}(\omega) = \sum_{i \in \omega} \tilde{b}_i$. If $\tilde{v}(\omega) < \tilde{v}(\tilde{\omega})$, then bidders cannot be given the discounts $d^\star$ calculated in Step d), as there exists a different combination of assignment bids that could provide greater revenues if assignment bids were modified by these discounts. A new constraint is required; proceed to Step h).

Otherwise, the maximum discount that can be granted to bidders is equal to $\sum_{j \in N} d_j^\star = D^\star$. Go to Step i).

h) If there are bidders from the original winner determiantion procedure whose assignment option given the modified assignment bids is not the same as the option resulting form the original assignment bids, and whose modfied assignment bid is not positive,[1] then add the following constraint to the list of constraints C:

$$\sum_{j \in L} d_j \leq \sigma(L)$$

where L denotes the set of bidders who receive a different assignment option and whose modified assignment bid is not positive. Go to Step d).

[1] This reflects the fact that the "losers" are those bidders whose preferences for certain assignment options are not taken into account. This means that where bidders receive an assignment option that is different from the option they receive with their originaly assignment bid because their modified assignment bid is zero, those bidders were granted too large a discount.

i) If there were a single solution to the optimisation problem in Step d) these discounts can be used to calculate additional prices. Proceed to Step n).

Otherwise, pursuant to the fourth condition in Rule 4.8.2, we will need to find a combination of discounts that distributes the total available surplus (i.e. the total available discount) evenly across bidders relative to the maximum surplus calculated for each bidder. This is done by minimising the sum of squares of the difference between the maximum surplus and the actual surplus (equal to the bid discount) applied in calculating additional prices for each bidder: $\min \sum_{j \in N} (d_j - \sigma(j))^2$, subject to the constraints in C and the constraint that the sum of the individual discounts by bidders must be equal to the maximum total discount that can be granted to bidders ($D^\star$) as calculated in Step g). Let $d^{\star\star}$ be the solution to this minimisation problem.

j) For all bidders $j \in N$ reduce the bid amounts for all assignment bids submitted by Bidder j by $d_j^{\star\star}$; if this yields a negative bid amount, set this bid amount to zero. Therefore, the modified assignment bids are defined as $\tilde{b}_i = \max\left[b_i - d_j^{\star\star}, 0\right]$ for $i \in B_j, j \in W$, $\tilde{b}_i = b_i$ for $i \in B_j, j \notin W$, giving together a set of modified assignment bids $\tilde{B}$.

k) Solve the winner determination problem when using $\tilde{B}$. This yields $\tilde{\omega}$ and $\tilde{v}(\tilde{\omega})$.

l) Calculate the sum of bid amounts of the original winning assignment bids using the reduced bid amounts, $\tilde{v}(\omega) = \sum_{i \in \omega} \tilde{b}_i$. If $\tilde{v}(\omega) < \tilde{v}(\tilde{\omega})$, then winners cannot be given the discounts $d_j^{\star\star}$ calculated in Step k). A new constraint is required; proceed to Step m).

Otherwise, the discounts $d_j^{\star\star}$ calculated in step k) can be used to calculate additional prices. Proceed to Step n).

m) If there are bidders from the original assignment winner determination procedure whose assignment option given the modified assignment bids is not the same as the option resulting from the original assignment bids and whose modified assignment bid is not positive, then expand the list of constraints to include the following constraint:

$$\sum_{j \in L} d_j \leq \sigma(L)$$

where L denotes the set of bidders who receive a different assignment option and whose modified assignment bid is not positive Go to Step i).

n) The additional price for each bidder is the bid amount of its winning assignment bid reduced by the discount calculated in the previous step, i.e. $p_j = b_{\omega_j} - d_j^\star$ or $p_j = b_{\omega_j} - d_j^{\star\star}$ respectively for all $j \in N$.