



Worked Example - CCA

Addendum to the invitation to tender for frequency blocks for
the national provision of mobile telecommunication services in
Switzerland

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1 Introduction

1. In this note, we provide a fully worked example of the CCA which includes the primary rounds, supplementary round and assignment round. In addition, we also provide two more examples to illustrate how the different conditions in the definition of base prices are applied in practice.
2. To ease understanding, all examples contain only two categories, 800MHz and 900MHz. The reserve prices and eligibility points used are as set out in the main auction rules.

2 Worked example CCA

2.1 Setup

3. In this example, there are four bidders competing for spectrum in the 800MHz, and 900MHz band. There are a total of six 2x5MHz lots available in the 800MHz band and seven 2x5MHz lots in the 900MHz band. The reserve price for a lot in each category is CHF21.3 million. In both categories a single lot is associated with six eligibility points.
4. We assume further, that bidders have valuations as shown in Table 1 and bid straightforwardly on the basis of these valuations in the principal stage.

Table 1: Valuations – Worked example

Bidder	Package (lots)		Valuation in million CHF
	800MHz	900MHz	
Andre	1	4	500
Andre	2	3	450
Andre	2	1	300
Ben	1	4	600
Ben	0	4	550
Caroline	3	0	400
Donald	2	0	250

5. Each bidder has an initial eligibility that determines the maximum size of the package that they can bid on in the first round of the auction. For this example, we assume that all bidders have sufficient initial eligibility to bid for all the packages described in Table 1, and that everybody is able to bid for their largest wanted package at the reserve prices.
6. We assume further that bidders bid on the package that maximises their surplus, which we will simply define as the difference between their valuation of a package and the cost of it at current round prices. In the following, we assume that all bidders bid according to this strategy.

2.2 Primary rounds

7. In round 1, bidding begins at the reserve prices specified above. The following table gives an overview of the surplus calculations for each bidder. The package that provides the highest surplus at round prices is highlighted in orange for each bidder.

Table 2: Surplus calculation - First primary round

Bidder	Package (lots)		Valuation in CHF million	Cost in CHF million	Surplus in CHF million
	800MHz	900MHz			
Andre	1	4	500	106.5	393.5
	2	3	450	106.5	343.5
	2	1	300	63.9	236.1
Ben	1	4	600	106.5	493.5
	0	4	550	85.2	464.8
Caroline	3	0	400	63.9	336.1
Donald	2	0	250	42.6	207.4

8. Assuming that each bidder bids on the package that provides the greatest surplus, aggregate demand for 800MHz lots is 7 ($= 1 + 1 + 3 + 2$) and aggregate demand for 900MHz is 8 ($= 4 + 4$). Bids, total demand and activity are summarised in the following table.

Table 3: Primary bid round 1 results

Price per 800MHz lot: CHF21.3 million; Price per 900MHz lot: CHF21.3 million						
	Andre	Ben	Caroline	Donald	Total demand	Total supply
800MHz lots	1	1	3	2	7	6
900MHz lots	4	4	0	0	8	7
Bid (CHF million)	106.5	106.5	63.9	42.6		
Activity (defines eligibility for round 2)	30	30	18	12		

9. There is excess demand for both categories; therefore prices are increased for both. Suppose that the auctioneer raises prices for both categories to CHF36.5 million. All bidders continue bidding on the same packages as in the first round (as these still provide the greatest surplus) and thus all maintain their eligibility.

Table 4: Primary bid round 2 results

Price per 800MHz lot: CHF36.5 million; Price per 900MHz lot: CHF36.5 million						
	Andre	Ben	Caroline	Donald	Total demand	Total supply
800MHz lots	1	1	3	2	7	6
900MHz lots	4	4	0	0	8	7
Bid (CHF million)	182.5	182.5	109.5	73		
Activity (defines eligibility for round 3)	30	30	18	12		

10. There is excess demand in both categories in round 2. The auctioneer decides to increase the price for both categories to CHF54.8 million. At these prices it becomes more profitable for Ben to bid on four 900MHz lots only and to stop bidding on 800MHz. Bidding on a package of four 900MHz lots gives him a surplus of CHF330.8 million (CHF550 million – CHF219.2 million) whereas bidding on a package of one 800MHz lot and four 900MHz lots would only give him surplus of CHF326 million (CHF600 million – CHF274 million) at these prices. Hence, he only bids on four 900MHz lots in this round. The results of round 3 are shown in the following table.

Table 5: Primary bid round 3 results

Price per 800MHz lot: CHF54.8 million; Price per 900MHz lot: CHF54.8 million						
	Andre	Ben	Caroline	Donald	Total demand	Total supply
800MHz lots	1	0	3	2	6	6
900MHz lots	4	4	0	0	8	7
Bid (CHF million)	274	219.2	164.4	109.6		
Activity (defines eligibility for round 4)	30	24	18	12		

11. This means that there is now only excess demand for 900MHz. The auctioneer decides to increase the price for 900MHz to CHF82.2 million. At these prices, the optimal packages remain unchanged for each bidder, and the results are shown in the following table.

Table 6: Primary bid round 4 results

Price per 800MHz lot: CHF54.8 million; Price per 900MHz lot: CHF82.2 million						
	Andre	Ben	Caroline	Donald	Total demand	Total supply
800MHz lots	1	0	3	2	6	6
900MHz lots	4	4	0	0	8	7
Bid (CHF million)	383.6	328.8	164.4	109.6		
Activity (defines eligibility for round 5)	30	24	18	12		

12. There is thus again excess demand for 900MHz lots, but not for 800MHz lots. For round 5, the auctioneer decides to increase the price for 900MHz to CHF102.8 million. At these prices, it is most profitable for Andre to bid on a package containing two 800MHz lots and one 900MHz lot giving him a surplus of CHF87.6 million (CHF300 million - CHF212.4 million). Bidding on a package containing one 800MHz lot and four 900MHz lots would only provide him with surplus of CHF34 million (CHF500 million - CHF466 million). Also, bidding on a package containing two 800MHz lots and three 900MHz lots would only provide him with surplus of CHF32 million (CHF450 million - CHF418 million). Hence, he bids on a package of two 800MHz and one 900MHz lots as this is his most profitable package at these prices. All other bidders continue bidding on the same packages as in round 4 as these are still there most profitable packages. The resulting bid decisions are shown in the following table.

Table 7: Primary bid round 5 results

Price per 800MHz lot: CHF54.8 million; Price per 900MHz lot: CHF102.8 million						
	Andre	Ben	Caroline	Donald	Total demand	Total supply
800MHz lots	2	0	3	2	7	6
900MHz lots	1	4	0	0	5	7
Bid (CHF million)	212.4	411.2	164.4	109.6		
Activity	18	24	18	12		

13. At these prices, there is now excess demand for 800MHz lots and excess supply for 900MHz lots. The auctioneer decides to raise the price for 800MHz to CHF82.2 million per lot. At the resulting prices, the bidders' most profitable packages are the same as in round 5. The resulting bid decisions are shown in Table 8.

Table 8: Primary bid round 6 results

Price per 800MHz lot: CHF82.2 million; Price per 900MHz lot: CHF102.8 million						
	Andre	Ben	Caroline	Donald	Total demand	Total supply
800MHz lots	2	0	3	2	7	6
900MHz lots	1	4	0	0	5	7
Bid (CHF million)	267.2	411.2	246.6	164.4		
Activity	18	24	18	12		

14. At the prices in round 6, there is thus excess demand for lots in the 800MHz category and excess supply for lots in the 900MHz category. The auctioneer decides to raise the price for lots in the 800MHz category to CHF102.8 million. At the resulting prices, all of Andre's packages are unprofitable to bid on as the package prices are above the respective valuations. He therefore drops demand in both categories to zero. The other bidders' most profitable packages at these prices are the same as in round 6. The resulting bid decisions are summarised in Table 9.

Table 9: Primary bid round 7 results

Price per 800MHz lot: CHF102.8 million; Price per 900MHz lot: CHF102.8 million						
	Andre	Ben	Caroline	Donald	Total demand	Total supply
800MHz lots	0	0	3	2	5	6
900MHz lots	0	4	0	0	4	7
Bid (CHF million)	0	411.2	308.4	205.6		
Activity	0	24	18	12		

15. There is no excess demand in either category in round 7, and thus no further primary round takes place. The auction proceeds to the supplementary round.

2.3 Supplementary round

16. The supplementary round presents the bidders an opportunity to express:
- their demand for packages of bids they did not bid for during the primary bids round; and
 - their maximum willingness to pay for packages that they did bid on in the primary bid rounds.
17. Supplementary bids will be subject to constraints depending on the bids made by a bidder in the primary bids round. Note that the electronic auction system will calculate

these constraints for bidders and it will not be possible to submit bids that do not comply with the constraints.

18. For each bidder, we identify their 'final primary bid package'. This is the most recent, non-zero primary bid submitted by that bidder. If a bidder submitted this bid in the last primary round, then its supplementary bid amount for this package is uncapped. If the final primary bid package had been submitted in any earlier round, then the supplementary bid for this package would be capped to the amount that results from applying the prevailing prices for the component lots in the last primary round when the bidder would have been eligible to bid for this package (i.e. the round immediately after the one in which they submitted their final primary bid package).
19. All other supplementary bids are constrained by the following rule: For any package A (other than the final primary package), an anchor package is determined as follows. Suppose the bidder was last eligible to bid for A in round n, but that in round n, the bidder made a primary bid for package B with a lower sum of eligibility points across lots. Then package B is the anchor package, and the supplementary bid for A cannot exceed the bidder's highest bid for B plus the price difference between A and B at the prices of round n. Note that the bidder's highest bid for B could be either a supplementary bid or a primary bid.
20. Thus, all caps in the supplementary bids round are effectively linked back to the supplementary bid for the final primary bid package:
 - Caps for all supplementary bids can be increased if the supplementary bid for the final package is increased.
 - If the bidder is still active at the end of the primary rounds, only the relative amounts that can be bid for different packages are restricted, but not the absolute bid amounts.
21. The implications of this rule are illustrated for all bidders in Table 10, which shows the calculation of the caps on supplementary bids for all packages for which the respective bidder has a valuation, and the supplementary bids that the bidder submits in the round. Final primary bids, which are the basis for calculating the caps (where relevant) are highlighted in blue.

Table 10: Supplementary bid constraints and supplementary bids

Bidder	Package (lots)		Valuation in CHF million	Highest primary bid in CHF million	Relative cap for supplementary bid	Supplementary bid in CHF million
	800MHz	900MHz				
Andre	1	4	500	383.6	CHF553.6m Highest Bid on (2,1) + price difference between (1,4) and (2,1) in round 5 (CHF253.6m)	500
	2	3	450	-	CHF505.6m Highest Bid on (2,1) + price difference between (2,3) and (2,1) in round 5 (CHF205.6m)	450
	2	1	300	267.2	CHF308.4m Constrained by the prices in round 7	300
Ben	1	4	600	182.5	CHF604.8m Highest bid on (0,4) + price difference between (1,4) and (0,4) in round 3 (CHF54.8m)	600
	0	4	550	411.2	Unconstrained because this is his final primary round package	550
Caroline	3	0	400	308.4	Unconstrained because this is his final primary round package	400
Donald	2	0	250	205.6	Unconstrained because this is his final primary round package	250

2.4 Winner determination in principal stage

22. All of the bids in Table 11 represent the maximum amount that each bidder is willing to pay for a specific package of lots. A winner determination process selects the combination of bids with the highest value that can be accommodated with the given spectrum, subject that at most one bid from each bidders is selected. The following table shows all supplementary bids and the winning packages, highlighted in orange. This combination of bids produce the highest value of CHF1.45 billion amongst all the possible combinations that could be accommodated with the available spectrum.

Table 11: Bids considered for winner determination in principal stage

Bidder	Package (lots)		Maximum bid (CHF million)	Primary or supplementary bid?
	800MHz	900MHz		
Andre	1	4	500	SB
	2	3	450	SB
	2	1	300	SB
Ben	1	4	600	SB
	0	4	550	SB
Caroline	3	0	400	SB
Donald	2	0	250	SB

2.5 Base price determination at the end of the principal stage

2.5.1 Second prices and opportunity cost

23. The price each winner pays for winning a certain package of generic lots is called the base price. This will be calculated using a second price rule which is designed to make sure that winners pay the minimum amount they could have paid and still won. Put differently, the pricing algorithm performs automatic bid shading on behalf of bidders down to the minimum amount that is necessary for winning – and does so without creating any risk for bidders as this bid shading happens with full knowledge of all the bids submitted. This means that if all the winners had hypothetically bid their base price rather than their winning bid (and had each lowered their bids for any other packages by the difference between winning bid and base price in order to retain relative valuation differences in their bid structure), then they would still have won.¹ Any larger reduction in their bids would have led to at least some of them losing. Therefore, the base price is the minimum amount each winner would have needed to bid in order not to lose.
24. Base prices are a **generalised** notion of the opportunity cost caused by winners. The opportunity cost of an individual bidder's bids is the highest value that could have been realised if the lots won by that bidder had been available to give to somebody else, and this notion of opportunity cost is reflected in the **Vickrey price**. It is determined by asking what the winning allocation would have been if we had ignored all the bids from a particular bidder. For example, suppose that the total value of all winning bids is 1000 with one particular bidder winning a set of lots with a bid of 100. If we now hypothetically eliminated this bidder and recalculated the winning outcome,

¹ Strictly speaking, we should say they 'could' still have won, as at base prices the winning bids may be one of a number of tied optimal allocations.

the total value of winning bids will in most cases be lower.² It might fall by the amount of the eliminated bid, namely in the case where the lots won by the bidder in question could not have been sold to any other bidder or group of bidders. In this case, giving the lots to that bidder entails no opportunity cost, and the Vickrey price that this bidder would have to pay would be zero. In most cases, it will be possible to accommodate some other bidder (or group of bidders), however, and thus the fall in value will be less dramatic, say to 960. In this case, the value that could have been created if the bidder in question had not been admitted is 60, i.e. the value of the excluded bid less the fall in the total value of bids that would result from exclusion.

25. The base price extends this notion of opportunity cost. It considers not just the case of hypothetically eliminating one winning bidder and asking what value could be created for other bidders, but also considers the opportunity cost of excluding subsets of winners. In order to ensure that no alternative combination of bidders would have produced a higher bid value, it must be the case that each subset of winners *together* pay the opportunity cost they jointly cause. Base prices will often be higher than Vickrey prices (but can never be lower).
26. Base prices have the property that total revenue is minimised subject to constraints that each winner and group of winners pays a sufficient amount that there are no unhappy losers. In some cases, this is insufficient to define a unique set of base prices, as there may be situations in which two or more winners together need to pay a certain amount, but there is some flexibility about how this is split amongst the winners. In this case, a further “equal splitting” rule resolves the ambiguity by picking a set of base prices that is as close as possible to the Vickrey prices. The examples in the second part of this document show how this rule would be applied.

2.5.2 Determining base prices in the worked example

27. The explanation is linked to the algorithmic description of how base prices are determined that is annexed to the main auction rules, following the individual steps in the order set out there. Where we use mathematical notation, this corresponds to the respective notation in this algorithmic description.
28. As a reminder, as shown in Table 11, the value-maximising combination of winning bids is:
 - Andre’s bid for two 800MHz lots and three 900MHz lots (with a bid amount of CHF450 million);
 - Ben’s bid for one 800MHz lot and four 900MHz lots (with a bid amount of CHF600 million);
 - Caroline’s bid for three 800MHz lots (with a bid amount of CHF400 million).

The total value of winning bid amounts is CHF1.45 billion.

a) For each winning bidder, calculate the sum of the bid amounts that would result from the winner determination procedure if that bidder had not submitted a bid.

b) Determine the minimum bids and the maximum discounts.

² This is not necessarily the case, however, if there are tied optimal combinations of winning bids, and the excluded bidder is not part of another tied optimal combination. In this case, excluding the winning bidder will simply mean that another combination of bids with the same value could be selected.

29. Maximum discount for Andre:

- *Maximum surplus:* If we discard all bids submitted by Andre, the next best allocation would include Ben's and Caroline's original winning bids, and Donald's bid for two 800MHz lots (with a bid amount of CHF250 million). The value of this combination of bids is CHF1.25 billion. Therefore, Andre's maximum surplus is CHF200 million (the original sum of winning bids of CHF1.45 billion – CHF1.25 billion).
- *Maximum discount to reserve:* The reserve price for Andre's winning package is CHF106.5 million (i.e. $5 * CHF21.3$ million), and the maximum discount to reserve is Andre's winning bid amount (CHF450 million) minus the reserve, yielding CHF343.5 million.
- *Maximum discount:* The maximum discount that could be granted to Andre is the minimum of CHF200 million and CHF343.5 million, which is CHF200 million.

30. Maximum discount for Ben:

- *Maximum surplus:* If we discard all bids submitted by Ben, the next best allocation would include Andre's bid for one 800MHz lot and four 900MHz lots (with a bid amount of CHF500 million), Caroline's original winning bid and Donald's bid for two 800MHz lots (with a bid amount of CHF250 million). The value of this combination of bids is CHF1.15 billion. Therefore, Ben's maximum surplus is CHF300 million.
- *Maximum discount to reserve:* The reserve price for Ben's winning package is CHF106.5 million, and the maximum discount to reserve is Ben's winning bid amount (CHF600 million) minus the reserve, yielding CHF493.5 million.
- *Maximum discount:* The maximum discount that could be granted to Ben is the minimum of CHF300 million and CHF493.5 million, which is CHF300 million.

31. Maximum discount for Caroline:

- *Maximum surplus:* If we discard all bids submitted by Caroline, the next best allocation would include Andre's and Ben's original winning bids and Donald's package bid for two 800MHz lots (with a bid amount of CHF250 million). The value of this combination of bids is CHF1.3 billion. Therefore, Caroline's maximum surplus is CHF150 million.
- *Maximum discount to reserve:* The reserve price for Caroline's winning package is CHF63.9 million, and the maximum discount to reserve is Caroline's winning bid amount (CHF400 million) minus the reserve, yielding CHF336.1 million.
- *Maximum discount:* The maximum discount that could be granted to Caroline is the minimum of CHF150 million and CHF336.1 million, which is CHF150 million.

32. The calculation of maximum discount for each winner is summarised in Table 12.

Table 12: Maximum discounts and initial lower bounds for base prices

Bidder	Maximum value of winning bids if bidder excluded	Calculation of maximum discount that can be granted to this winner	Initial lower bound for the base price to be paid by this winner
Andre	CHF1.25b	<ul style="list-style-type: none"> ▪ σ (Andre) = CHF1.45b – CHF1.25b = CHF200m ▪ $m_{\text{Andre}} = \text{CHF}106.5\text{m}$ ▪ $\rho_{\text{Andre}} = \text{CHF}450\text{m} - \text{CHF}106.5\text{m} = \text{CHF}343.5\text{m}$ $d_{\text{Andre}}^{\max} = \min(\text{CHF}200\text{m}, \text{CHF}343.5\text{m}) = \text{CHF}200\text{m}$	CHF250m (CHF450m – CHF200m)
Ben	CHF1.15b	<ul style="list-style-type: none"> ▪ σ (Ben) = CHF1.45b – CHF1.15b = CHF300m ▪ $m_{\text{Ben}} = \text{CHF}106.5\text{m}$ ▪ $\rho_{\text{Ben}} = \text{CHF}600\text{m} - \text{CHF}106.5\text{m} = \text{CHF}493.5\text{m}$ $d_{\text{Ben}}^{\max} = \min(\text{CHF}300\text{m}, \text{CHF}493.5\text{m}) = \text{CHF}300\text{m}$	CHF300m (CHF600m – CHF300m)
Caroline	CHF1.3b	<ul style="list-style-type: none"> ▪ σ (Caroline) = CHF1.45b – CHF1.3b = CHF150m ▪ $m_{\text{Caroline}} = \text{CHF}63.9\text{m}$ ▪ $\rho_{\text{Caroline}} = \text{CHF}400\text{m} - \text{CHF}63.9\text{m} = \text{CHF}336.1\text{m}$ $d_{\text{Caroline}}^{\max} = \min(\text{CHF}150\text{m}, \text{CHF}336.1\text{m}) = \text{CHF}150\text{m}$	CHF250m (CHF400m – CHF150m)

c) Generate a list of constraints and initialise it with the conditions that:

(i) for each winner, the individual discount cannot be negative (i.e., bidders cannot be asked to pay a base price that is greater than their bid amount);

(ii) for each winner, the individual discount is not greater than the maximum discount calculated in Step b); and

(iii) the aggregate discount over all winners is not greater than the reduction in the sum of bid amounts if all the bids from all winners were discarded.

33. If all bids from winning bidders were discarded, Donald would win two 800MHz lots. The value of this allocation would be CHF250 million, and the maximum sum of discounts that could potentially be granted is thus CHF1.2 billion (CHF1.45 billion - CHF250 million).

34. Therefore, the list of initial constraints (C) contains the following:

$$d_{\text{Andre}} \geq 0,$$

$$d_{\text{Ben}} \geq 0,$$

$$d_{Caroline} \geq 0,$$

$$d_{Andre} \leq \text{CHF}200\text{m},$$

$$d_{Ben} \leq \text{CHF}300\text{m},$$

$$d_{Caroline} \leq \text{CHF}150\text{m},$$

$$d_{Andre} + d_{Ben} + d_{Caroline} \leq \text{CHF}1.2\text{b}$$

d) Maximise the total discount given to winners subject to the constraints in C.

35. The solution to this maximisation problem given the constraints is unique and given by:

$$d_{Andre}^* = \text{CHF}200\text{m},$$

$$d_{Ben}^* = \text{CHF}300\text{m},$$

$$d_{Caroline}^* = \text{CHF}150\text{m}$$

e) Reduce the bid amounts for all winning bidders by the discounts found in Step d); if this yields a negative bid amount, set the bid amount to zero.

36. Reducing the bid amounts of all bids made by winning bidders by the respective discounts obtained in Step d), we obtain the set of bids illustrated in Table 13.

f) Determine the winning bids when replacing all bids by the reduced bid amounts and calculate the corresponding sum of total winning bid amounts for this new set of winning bids.

37. The winning combination using the reduced bid amounts is the same as the original winning combination.³ The value of this allocation (using the reduced bid amounts) is CHF800 million.

³The original combination of winning bids achieves the same highest value as three other combinations. The three other combinations are: Ben (1,4), Caroline (3,0) and Donald (2,0); Andre (1,4), Caroline (3,0) and Donald (2,0); and Andre (2,3), Ben (1,4) and Donald (2,0).

Table 13: Modified package bids using reduced bid amounts

Bidder	Bid package (lots)		Bid amount
	800MHz	900MHz	
Andre	1	4	CHF300m (CHF500m – CHF200m)
	2	3	CHF250m (CHF450m – CHF200m)
	2	1	CHF100m (CHF300m – CHF200m)
Ben	1	4	CHF300m (CHF600m – CHF300m)
	0	4	CHF250m (CHF550m – CHF300m)
Caroline	3	0	CHF250m (CHF400m – CHF150m)
Donald	2	0	CHF250m

g) If the sum of bid amounts of the original winning bids (using the reduced bid amounts) is smaller than the sum of winning bid amounts for the new set of winning bids calculated in Step f), then winners cannot be given the discounts calculated as there exists a different combination of bidders that could provide greater revenues given the bids submitted. A new constraint is required; proceed to Step h).

Otherwise, calculate the maximum discount that can be granted to winning bidders and go to Step i).

38. The winning combination of bids found in Step f) is the same as the original winning combination of bids, and therefore the original winning bids using the reduced bid amounts yield the same value as the solution found in Step f). We can therefore calculate the maximum total discount that can be granted to winning bidders:

$$D^* = \text{CHF}200\text{m} + \text{CHF}300\text{m} + \text{CHF}150\text{m} = \text{CHF}650\text{m}.$$

39. We now proceed to Step i).

i) If there is only one combination of discounts that provide the maximum total discount given to winners, these will be used in the calculation of base prices. Proceed to Step n).

Otherwise, we will identify a combination of discounts that distributes winner surplus evenly in relation to the maximum surplus calculated for each winner.

40. The maximum discount that can be granted to winning bidders can only be implemented using the maximum surplus for each winner; therefore, these provide the discounts that allow us to calculate final prices. Proceed to Step n).

n) The price for each winner is equal to the bid amount of its winning bid minus the discount.

41. We can proceed to calculate the base prices, shown in Table 14.

Table 14: Base prices for winning bids

Bidder	Bid	Discount (Surplus)	Base price
Andre	CHF450m	CHF200m	CHF250m
Ben	CHF600m	CHF300m	CHF300m
Caroline	CHF400m	CHF150m	CHF250m

42. In this example, Andre needs to pay enough in order to outbid Donald on two 800MHz lots and therefore needs to pay CHF250 million. The same is true for Caroline who also needs to pay that amount.
43. If Ben did not participate, Andre would win his package of one 800MHz and four 900MHz lots at a bid amount of CHF500 million, which is CHF50 million higher than his winning bid of CHF450 million for his winning package. In addition, Donald would win his package of two 800MHz lots. In order to outbid both of these bidders on his winning package, Ben has to pay CHF300 million.

2.6 Assignment stage

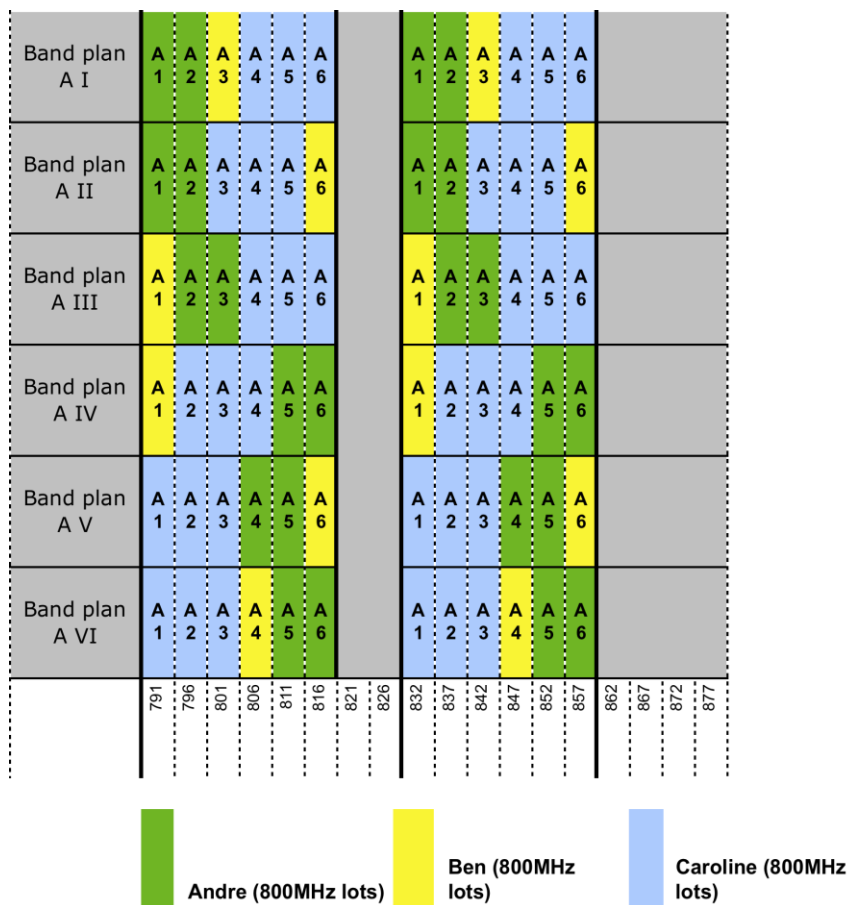
44. The principal stage will have determined how much spectrum bidders will receive, but not the frequency ranges. The purpose of the assignment stage is to determine how the available frequencies in each band are distributed amongst the winning bidders from the principal stage, and the final price to be paid by each winning bidder.
45. There is a separate assignment procedure for each spectrum band, although any bidding and frequency selection will take place simultaneously for both these bands. If there is only one winner in a band, then no further bidding is required. For bands where there are two or more winning bidders, an assignment stage is required.
46. It is possible that some or all lots in either band may be unassigned at the end of the principal stage. If there are some unassigned lots in a band, this may limit the frequency selection options for winning bidders in that band, as unassigned lots will be required to be contiguous with each other and placed according to predetermined conditions.

2.6.1 Assignment stage options

47. For each band where there are two or more winning bidders, the auctioneer will determine the set of frequency assignment options available to each bidder. For each bidder in each category, the auctioneer will identify the exhaustive list of packages of contiguous frequencies which are consistent with the number of lots that they won in the principal stage, while also allowing all other winners to receive contiguous frequency assignments and leaving any unassigned spectrum in a single contiguous frequency block. In practice, there are likely to be a relatively small number of such assignment options given the requirement for contiguity.
48. In this example, all abstract frequency blocks were auctioned off in the principal stage, with the following winners emerging from that stage:
- Andre won two 800MHz lots and three 900MHz lots;
 - Ben won one 800MHz lot and four 900MHz lots; and
 - Caroline won three 800MHz lots.

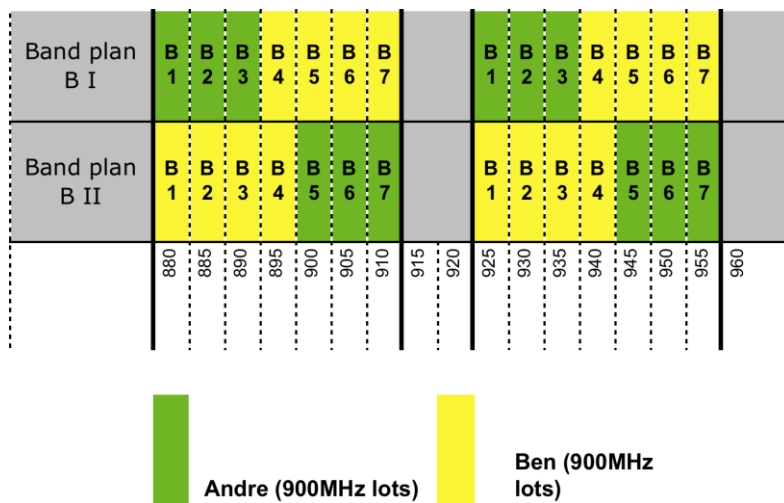
49. The assignment stage is subdivided into two simultaneous procedures:
- The assignment of specific 800MHz frequency blocks; and
 - The assignment of specific 900MHz frequency blocks.
50. The possible band plans for 800MHz spectrum are shown in Figure 1. They result in the following assignment options for each bidder:
- Andre: A1-A2; A2-A3; A4-A5; A5-A6.
 - Ben: A1; A3; A4; A6.
 - Caroline: A1-A3; A2-A4; A3-A5; A4-A6.

Figure 1: Possible band plans for 800MHz band



51. The possible band plans for 900MHz spectrum are shown in Figure 2. They result in the following assignment options for bidders:
- Andre: B1-B3; B5-B7.
 - Ben: B1-B4; B4-B7.

Figure 2: Possible band plans for 900MHz band



2.6.2 Assignment stage bids and winner determination

52. During the assignment stage, bidders submit assignment bids for specific assignment options. Each assignment option (which will be identified by the starting block of the assignment in the following description) is associated with one or more possible band plans. There are six different possible band plans (Band plans A I to A VI) for the 800MHz band as shown in Figure 1 and two possible band plans (Band plans B I and B II) for the 900MHz band as shown in Figure 2.
53. The following table shows the assignment bids made by bidders in the assignment round for the assignment of specific frequencies in the 800MHz band (zero bids are submitted automatically by the auction software). Equal assignment bids can be interpreted as indicating indifference between the assignment options, and preferences for particular assignment options over others are expressed through positive assignment bids.

Table 15: Assignment bids – 800MHz band

Bidder	Number of abstract frequency blocks acquired	Starting block of assignment option	Assignment bid in CHF million	Associated band plan(s)
Andre	2	A1	0.2	A I, A II
		A2	0.1	A III
		A4	0	A V
		A5	0.3	A IV, A VI
Ben	1	A1	0.5	A III, A IV
		A3	0	A I
		A4	0	A VI
		A6	0	A II, A V

Caroline	3	A1	0.8	A V, A VI
		A2	0	A IV
		A3	0	A II
		A4	0	A I, A III

54. The winning combination of assignment bids for the 800MHz band is shown in orange in the table above. As in the winner determination procedure in the principal stage, the highest combination of assignment bids that can be accommodated subject to every winner of this spectrum receiving contiguous spectrum is chosen. The total amount of assignment bids for each of the possible band plans is as follows:

- A I: CHF0.2 million;
- A II: CHF0.2 million;
- A III: CHF0.6 million;
- A IV: CHF0.8 million;
- A V: CHF0.8 million; and
- A VI: CHF1.1 million.

55. Hence, the winning band plan is A VI and the winning assignment bids for the 800MHz band are:

- Andre's assignment bid of CHF0.3 million for an assignment starting with the frequency block A5;
- Ben's assignment bid of CHF0 for an assignment starting with the frequency block A4; and
- Caroline's assignment bid of CHF0.8 million for an assignment starting with the frequency block A1.

56. The following table shows the assignment bids made in the assignment round for specific frequencies in the 900MHz band (zero bids are submitted automatically by the auction software).

Table 16: Assignment bids – 900MHz band

Bidder	Number of abstract frequency blocks acquired	Starting block of assignment option	Assignment bid in CHF million	Associated band plan(s)
Andre	3	B1	0.2	B I
		B5	0	B II
Ben	4	B1	0.5	B II
		B4	0	B I

57. The total amount of assignment bids for the two possible assignments of 900MHz spectrum are as follows:
- B I: CHF0.2 million; and
 - B II: CHF0.5 million.
58. Hence, assignment B II is selected and Andre is assigned the frequency blocks B5 to B7 and Ben is assigned the frequency blocks B1 to B4.

2.6.3 Determination of additional prices

Determination of additional prices for the specific assignments in the 800MHz band

59. The calculation of additional prices using the modified second-price rule is similar to the calculation of base prices in the principal stage. The difference is that in the calculation of opportunity costs (or the maximum discounts according to algorithmic description in the annex to the auction rules) not all of a bidder's assignment bids are eliminated; instead, the bid amounts are set to the respective minimum bid (in this case zero). The reason for this difference is that the opportunity costs are determined by a bidder's preference for certain actual frequency blocks and not by the bidder's demand for frequency blocks in general. The reduction of all bids to the respective minimum bids (i.e. zero) reflects the case in which a bidder expresses no preferences for any specific frequency assignment. Algorithmically, the procedure for establishing additional prices is set out in an annex to the auction rules, but we provide here a shortened description rather than running through the various individual steps.
60. For the 800MHz band, the winning bids are the assignment bids submitted by Andre, Ben and Caroline for assignment A VI (a zero bid was placed automatically for Ben). If all of Andre's assignment bids were set to zero, the assignment A VI would still win (in a tie with A V). Thus, if Andre did not express a preference for a specific assignment, Caroline would still be allocated the frequencies A1-A3 and Ben would either be allocated A4 or A6 with Andre being assigned the remaining frequencies A5-A6 or A4-A5, respectively. The revenue given this modified set of assignment bids is CHF0.8 million. Hence, the maximum discount that could be granted to Andre is CHF0.3 million (CHF1.1 million – CHF0.8 million).
61. If all of Ben's assignment bids were reduced to zero, the assignment A VI would still win and lead to the same revenue. Hence, the maximum discount that could be granted to Ben is CHF0 (CHF1.1 million – CHF1.1 million).
62. If all of Caroline's assignment bids were reduced to zero, then assignment option A IV would win, which leads to revenues of CHF0.8 million. Hence, if Caroline did not express a preference for a specific frequency assignment, Andre would be assigned A5-A6 and Ben would be assigned A1. In this case, Caroline would be allocated the remaining frequencies A1-A4. The revenue of this assignment is CHF0.8 million. Hence, the maximum discount that could be granted to Caroline is CHF0.3 million (CHF1.1 million – CHF0.8 million).
63. The results are summarised in Table 17 below.

Table 17: Calculation of potential additional prices for the assignment of specific frequencies in the 800MHz band

Bidder	Frequency blocks	Modified maximum revenue amount	Maximum discount	Potential additional price
Andre	A5-A6	CHF0.8m	CHF1.1m - CHF0.8m = CHF0.3m	CHF0.3m - CHF0.3m = CHF0
Ben	A4	CHF1.1m	CHF1.1m - 1.1m = CHF 0	CHF0 - CHF0 = CHF0
Caroline	A1-A3	CHF0.8m	CHF1.1m -CHF0.8m = CHF0.3m	CHF0.8m - CHF0.3m = CHF0.5m

64. The linear optimisation yields the discounts shown in Table 17. In the next, step, we subtract these discounts from the assignment bids to obtain the set of modified assignment bids. If this yields a negative bid amount, the bid is set to zero. The results of the winner determination for the modified assignment bids is shown in orange in Table 18.

Table 18: Modified assignment bids – 800MHz band

Bidder	Number of abstract frequency blocks acquired	Starting block of assignment option	Modified assignment bid in CHF million	Associated band plan(s)
Andre	2	A1	0	A I, A II
		A2	0	A III
		A4	0	A V
		A5	0	A IV, A VI
Ben	1	A1	0.5	A III, A IV
		A3	0	A I
		A4	0	A VI
		A6	0	A II, A V
Caroline	3	A1	0.5	A V, A VI
		A2	0	A IV
		A3	0	A II

65. Based on the modified assignment bids, the modified revenues associated with each band plan is as follows:
- A I: CHF0;
 - A II: CHF0;
 - A III: CHF0.5 million;
 - A IV: CHF0.5 million;
 - A V: CHF0.5 million; and
 - A VI: CHF0.5 million.
66. Band plans A III, A IV, A V and A VI all maximise modified revenues. As the original winning band plan A VI achieves the same highest value given the modified set of assignment bids, bidders can be granted the maximum discounts. These will thus be used to determine additional prices. In this example, additional prices are equal to the opportunity costs.

Determination of additional prices for the specific assignments in the 900MHz band

67. The additional prices for 900MHz are determined in a similar fashion. The assignment bids associated with assignment B II win which is a bid of CHF0.5 million from Ben to be assigned B1-B4 and a bid of CHF0 from Andre (added automatically by the system) to be assigned the frequency blocks B5-B7.
68. If Andre did not have a preference for any specific assignment and hence all his assignment bids were zero, then the assignment B II would still win with revenue of CHF0.5m. Hence the maximum discount that could be granted to Andre is CHF0 (CHF0.5 million – CHF0.5 million).
69. If Ben did not have a preference for any specific assignment and hence all his assignment bids were zero, then the assignment B I would win. This means Andre would be assigned B1-B3 with Ben being allocated B4-B7. The revenue associated with this assignment is CHF0.2 million. Hence the maximum discount that could be granted to Andre is CHF0.3 million (CHF0.5 million – CHF0.2 million).
70. The results are summarised in Table 19 below.

Table 19: Calculation of potential additional prices for the assignment of specific frequencies in the 900MHz band

Bidder	Frequency blocks	Modified maximum revenue amount	Maximum discount	Potential additional price
Andre	B5-B7	CHF0.5m	CHF0.5m - CHF0.5m = CHF0	CHF0 - CHF0 = CHF0

Ben	B1-B4	CHF0.2m	CHF0.5m - 0.2m = CHF0.3m	CHF0.5m - CHF0.3m = CHF0.2m
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71. The linear optimisation yields the discounts shown in Table 19. In the next step, we subtract these discounts from the assignment bids to obtain the set of modified assignment bids. If this yields a negative bid amount, the bid is set to zero. The results of the winner determination for the modified assignment bids are shown in orange in Table 20.

Table 20: Modified assignment bids – 900MHz band

Bidder	Number of abstract frequency blocks acquired	Starting block of assignment option	Modified assignment bid in CHF million	Associated band plan(s)
Andre	3	B1	0.2	B I
	3	B5	0	B II
Ben	4	B1	0.2	B II
	4	B4	0	B I

72. Based on the modified assignment bids, the modified revenues associated with each band plan is as follows:
- B I: CHF0.2 million; and
 - B II: CHF0.2 million.
73. Both band plans achieve the same modified revenue. As the original winning band plan B II achieves the same highest value given the modified set of assignment bids, bidders can be granted the maximum discounts. These will thus be used to determine additional prices. In this example, additional prices are equal to the opportunity costs.

2.7 Auction results

74. Adding the base prices determined in the principal stage (provided in Table 14) and the additional prices determined in the assignment stage (provided in Table 17 and Table 19) yields the overall price for each winning bidder. The results of the auction are shown in Table 21.

Table 21: Auction results

Bidder	Spectrum band	Number of lots won	Specific frequencies won	Base price in CHF million	Additional price in CHF million	Overall price in CHF million
Andre	800MHz	2	A5-A6: 811-820MHz/852-861MHz	250	0	250
	900MHz	3	B5-B7: 900-914MHz/945-959MHz		0	
Ben	800MHz	1	A4: 806-810MHz-847-851MHz	300	0	300.2
	900MHz	4	B1-B3+B4: 880-899MHz-925-944MHz		0.2	
Caroline	800MHz	3	A1-A3: 791-805MHz/832-846MHz	250	0.5	250.5

3 Additional base pricing examples

3.1 Overview

75. In the example provided in Section 2, it was feasible to grant winners their maximum discounts, and therefore the distribution of surplus between winners was straightforward. However, this is not possible in general, as often granting each bidder the maximum discount would yield base prices that are not high enough to ensure that no other combination of bidders could provide greater revenue.
76. In the following two examples, bidders pay base prices above their individual opportunity cost. In the example presented in Section 3.2, unique base prices are identified by the minimisation of the sum of base prices which is achieved by maximising the sum of discounts. In the example presented in Section 3.3, a coalition of winners needs jointly to outbid a third bidder. How this burden is split in terms of base prices paid by members of the coalition is determined by picking base prices that minimise the sum of squared differences between base prices and individual opportunity cost. In terms of the algorithmic implementation, this is achieved by picking discounts that minimise the sum of squared differences between the individual discounts and their respective maximum discounts.
77. The setup of these examples is the same as in the example presented in Section 2. We will however take a certain set of bids as given and only consider the winner and price determination procedure. We will not consider the assignment stage.

3.2 Example 2: Base prices uniquely identified by maximisation of discounts

78. In this example, base prices need to be greater than the base price that would result from giving each winner their maximum discount. However, there is a unique set of base prices that maximises the total discount over all winners subject to the constraints in Rule 3.3.32⁴, and therefore we do not need to choose base prices that distribute winner surplus in relation to the maximum surplus calculated for each winner evenly.
79. Bidders submit the bids shown in Table 22.

Table 22: Bids submitted – Example 2

Bidder	Bid package (lots)		Bid amount in CHF million
	800MHz	900MHz	
Andre	1	4	500
	2	3	475
Ben	0	4	700
Caroline	4	0	400

⁴ See auction rules.

Donald	0	3	250
	3	0	250
	1	4	850

80. The value-maximising combination of winning bids is:
- Andre's bid for two 800MHz lots and three 900MHz lots (with a bid amount of CHF475 million);
 - Ben's bid for four 900MHz lots (with a bid amount of CHF700 million);
 - Caroline's bid for four 800MHz (with a bid amount of CHF400 million).
81. The combination of winning bids is highlighted in orange in Table 22. The total value of winning bid amounts is CHF1.575 billion.
82. Again, we calculate base prices to be paid by each winner following the process described in the auction rules.
- a) For each winning bidder, calculate the sum of the bid amounts that would result from the winner determination procedure if that bidder had not submitted a bid.
- b) Determine the minimum bids and the maximum discounts.
83. Maximum discount for Andre:
- *Maximum surplus:* If we discard all bids submitted by Andre, the next best allocation would include Ben's and Caroline's original winning bids, and Donald's bid for three 900MHz lots (with a bid amount of CHF250 million). The value of this combination of bids is CHF1.35 billion. Therefore, Andre's maximum surplus is CHF225 million.
 - *Maximum discount to reserve:* The reserve price for Andre's winning package is CHF106.5 million, and the maximum discount to reserve is Andre's winning bid amount (CHF475 million) minus the reserve, yielding CHF368.5 million.
 - *Maximum discount:* The maximum discount that could be granted to Andre is the minimum of CHF225 million and CHF368.5 million, which is CHF225 million.
84. Maximum discount for Ben:
- *Maximum surplus:* If we discard all bids submitted by Ben, the next best allocation would include Andre's original winning bid and Donald's bid for one 800MHz lots and four 900MHz lots (with a bid amount of CHF850 million). The value of this combination of bids is CHF1.325 billion. Therefore, Ben's maximum surplus is CHF250 million.
 - *Maximum discount to reserve:* The reserve price for Ben's winning package is CHF85.2 million, and the maximum discount to reserve is Ben's winning bid amount (CHF700 million) minus the reserve, yielding CHF614.8 million.
 - *Maximum discount:* The maximum discount that could be granted to Ben is the minimum of CHF250 million and CHF614.8 million, which is CHF250 million.
85. Maximum discount for Caroline:
- *Maximum surplus:* If we discard all bids submitted by Caroline, the next best allocation would include Andre's and Ben's original winning bids and Donald's

package bid for three 800MHz lots (with a bid amount of CHF250 million). The value of this combination of bids is CHF1.425 billion. Therefore, Caroline's maximum surplus is CHF150 million.

- *Maximum discount to reserve:* The reserve price for Caroline's winning package is CHF85.2 million, and the maximum discount to reserve is Caroline's winning bid amount (CHF400 million) minus the reserve, yielding CHF314.8 million.
- *Maximum discount:* The maximum discount that could be granted to Caroline is the minimum of CHF150 million and CHF314.8 million, which is CHF150 million.

Table 23: Maximum discounts and initial lower bounds for base prices – Example 2

Bidder	Maximum value of winning bids if bidder excluded	Calculation of maximum discount that can be granted to this winner	Initial lower bound for the base price to be paid by this winner
Andre	CHF1.35b	<ul style="list-style-type: none"> ▪ σ (Andre) = CHF1.575b – CHF1.35b = CHF225m ▪ $m_{\text{Andre}} = \text{CHF}106.5\text{m}$ ▪ $\rho_{\text{Andre}} = \text{CHF}475\text{m} - \text{CHF}106.5\text{m} = \text{CHF}368.5\text{m}$ $d_{\text{Andre}}^{\text{max}} = \min(\text{CHF}225\text{m}, \text{CHF}368.5\text{m}) = \text{CHF}225\text{m}$	CHF250m (CHF475m – CHF225m)
Ben	CHF1.325b	<ul style="list-style-type: none"> ▪ σ (Ben) = CHF1.575b – CHF1.325b = CHF250m ▪ $m_{\text{Ben}} = \text{CHF}85.2\text{m}$ ▪ $\rho_{\text{Ben}} = \text{CHF}700\text{m} - \text{CHF}85.2\text{m} = \text{CHF}614.8\text{m}$ $d_{\text{Ben}}^{\text{max}} = \min(\text{CHF}250\text{m}, \text{CHF}614.8\text{m}) = \text{CHF}250\text{m}$	CHF450m (CHF700m – CHF250m)
Caroline	CHF1.425b	<ul style="list-style-type: none"> ▪ σ (Caroline) = CHF1.575b – CHF1.425b = CHF150m ▪ $m_{\text{Caroline}} = \text{CHF}85.2\text{m}$ ▪ $\rho_{\text{Caroline}} = \text{CHF}400\text{m} - \text{CHF}85.2\text{m} = \text{CHF}314.8\text{m}$ $d_{\text{Caroline}}^{\text{max}} = \min(\text{CHF}150\text{m}, \text{CHF}314.8\text{m}) = \text{CHF}150\text{m}$	CHF250m (CHF400m – CHF150m)

c) Generate a list of constraints and initialise it with the conditions that:

(i) for each winner, the individual discount cannot be negative (i.e., bidders cannot be asked to pay a base price that is greater than their bid amount);

(ii) for each winner, the individual discount is not greater than the maximum discount calculated in Step b); and

(iii) the aggregate discount over all winners is not greater than the reduction in the sum of bid amounts if all the bids from all winners were discarded.

86. If all bids from winning bidders were discarded, Donald would have won one 800MHz lot and four 900MHz lots. The value of this allocation would be CHF850 million, and hence the maximum sum of discounts that could potentially be granted is CHF725 million. (1.575 billion - CHF850 million).

87. Therefore, the initial list of constraints (C) contains the following:

$$d_{Andre} \geq 0,$$

$$d_{Ben} \geq 0,$$

$$d_{Caroline} \geq 0,$$

$$d_{Andre} \leq \text{CHF}225\text{m},$$

$$d_{Ben} \leq \text{CHF}250\text{m},$$

$$d_{Caroline} \leq \text{CHF}150\text{m},$$

$$d_{Andre} + d_{Ben} + d_{Caroline} \leq \text{CHF}725\text{m}$$

d) Maximise the total discount given to winners subject to the constraints in C.

88. The solution to this maximisation problem given the constraints is:

$$d_{Andre}^* = \text{CHF}225\text{m},$$

$$d_{Ben}^* = \text{CHF}250\text{m},$$

$$d_{Caroline}^* = \text{CHF}150\text{m}$$

e) Reduce the bid amounts for all winning bidders by the discounts found in Step d); if this yields a negative bid amount, set the bid amount to zero.

89. Reducing the bid amounts of all bids made by winning bidders by their respective discounts obtained in Step d), we obtain the set of bids illustrated in Table 24.

Table 24: Modified package bids using reduced bid amounts, first iteration – Example 2

Bidder	Bid Package (lots)		Bid amount
	800MHz	900MHz	
Andre	1	4	CHF275m (CHF500 - CHF225m)
	2	3	CHF250m (CHF475m – CHF225m)
Ben	0	4	CHF450m (CHF700m – CHF250m)
Caroline	4	0	CHF250m (CHF400m – CHF150m)
Donald	0	3	CHF250m

	3	0	CHF250m
	1	4	CHF850m

f) Determine the winning bids when replacing all bids by the reduced bid amounts and calculate the corresponding sum of total winning bid amounts for this new set of winning bids.

90. We find the combination that maximises revenues (using the reduced bid amounts), as highlighted in orange in Table 24. The value of this combination (using the reduced bid amounts) is CHF1.1 billion.

g) If the sum of bid amounts of the original winning bids (using the reduced bid amounts) is smaller than the sum of winning bid amounts for the new set of winning bids calculated in Step f), then winners cannot be given the discounts calculated as there exists a different combination of bidders that could provide greater revenues given the bids submitted. A new constraint is required; proceed to Step h).

91. The sum of the reduced bid amounts of the original winning bids (shown in bold font in Table 24) is CHF950 million, which is smaller than the value of the outcome calculated in Step f). We therefore need a new constraint, so we proceed to Step h).

h) If any of the original winners is no longer among the new winning bidders calculated in Step f), then add the corresponding constraint in C.

92. Caroline and Ben, who are winners in the original outcome, are no longer among the winning bidders calculated in Step f). To add the additional constraint to C, we need to calculate the joint maximum surplus of these two winners.

93. If all the bids submitted by Caroline and Ben were discarded, the winning combination (using the original unmodified bids) would be:

- Andre's bid for two 800MHz and three 900MHz lots (with a bid amount of CHF475 million);
- Donald's bid for one 800MHz lots and four 900MHz lots (with a bid amount of CHF850 million);

94. The total value of winning bid amounts in this case would be CHF1.325 billion.

Therefore, the maximum surplus that can be enjoyed Ben and Caroline taken together is

$$\sigma(\text{Ben}, \text{Caroline}) = \text{CHF}1.575\text{b} - \text{CHF}1.325\text{b} = \text{CHF}250\text{m}$$

and hence the maximum discount that can be granted to Ben and Caroline jointly cannot exceed CHF250 million. We add the following constraint to the list of constraints:

$$d_{\text{Ben}} + d_{\text{Caroline}} \leq \text{CHF}250\text{m}$$

95. We now re-run Step d).

d) Maximise the total discount given to winners subject to the constraints in C.

96. One of the solutions to the maximisation problem given the new list of constraints is:

$$d_{\text{Andre}}^* = \text{CHF}225\text{m},$$

$$d_{\text{Ben}}^* = \text{CHF}250\text{m},$$

$$d_{\text{Caroline}}^* = \text{CHF}0\text{m}$$

97. Note that there is only one possible solution. Any combination of non negative discounts for Ben and Caroline such that $d_{Ben} + d_{Caroline} = \text{CHF}250\text{m}$, $d_{Ben} \leq d_{Ben}^{max}$, $d_{Caroline} \leq d_{Caroline}^{max}$ and $d_{Andre} = d_{Andre}^{max}$ maximises the total discount given to winners. In this case, one of these solutions is picked at random.

e) Reduce the bid amounts for all winning bidders by the discounts found in Step d); if this yields a negative bid amount, set the bid amount to zero.

98. Reducing the bid amounts of all bids made by winning bidders by their respective discounts obtained in Step d), we obtain the set of bids illustrated in Table 25.

Table 25: Modified package bids using reduced bid amounts, second iteration – Example 2

Bidder	Bid package (lots)		Bid amount
	800MHz	900MHz	
Andre	1	4	CHF275m (CHF500m – CHF225m)
	2	3	CHF250m (CHF475m – CHF225m)
Ben	0	4	CHF450m (CHF700m – CHF250m)
Caroline	4	0	CHF400 (CHF400m - CHF0)
Donald	0	3	CHF250m
	3	0	CHF250m
	1	4	CHF850m

f) Determine the winning bids when replacing all bids by the reduced bid amounts and calculate the corresponding sum of total winning bid amounts for this new set of winning bids.

99. We find the combination that maximises revenues (using the reduced bid amounts), which is highlighted in orange in Table 25. The value of this allocation (using the reduced bid amounts) is CHF1.25 billion.

g) If the sum of bid amounts of the original winning bids (using the reduced bid amounts) is smaller than the sum of winning bid amounts for the new set of winning bids calculated in Step f), then winners cannot be given the discounts calculated as there exists a different combination of bidders that could provide greater revenues given the bids submitted. A new constraint is required; proceed to Step h).

100. The sum of the reduced bid amounts of the original winning bids (shown in bold font in Table 25) is CHF1.1 billion, which is smaller than the value of the outcome calculated in Step f). We therefore need a new constraint, so we proceed to Step h).

h) If any of the original winners is no longer among the new winning bidders calculated in Step f), then add the corresponding constraint in C.

101. Andre and Ben, who are winners in the original outcome, are no longer among the new winning bidders calculated in Step f). To add the additional constraint to C, we need to calculate the joint maximum surplus of these two winners.

102. If all the bids submitted by Andre and Ben were discarded, the winning combination (using the original unmodified bids) would be:

- Caroline's bid for four 800MHz lots (with a bid amount of CHF400 million);
- Donald's bid for one 800MHz lots and four 900MHz lots (with a bid amount of CHF850 million);

103. The total value of winning bid amounts in this case would be CHF1.25 billion.

Therefore, the maximum surplus of Ben and Andre taken together is

$$\sigma(\text{Andre}, \text{Ben}) = \text{CHF}1.575\text{b} - \text{CHF}1.25\text{b} = \text{CHF}325\text{m}$$

and hence the maximum discount that can be granted to Andre and Ben jointly cannot exceed CHF325 million. We add the following constraint to the list of constraints:

$$d_{\text{Andre}} + d_{\text{Ben}} \leq \text{CHF}325\text{m}$$

d) Maximise the total discount given to winners subject to the constraints in C.

104. The solution to the maximisation problem given the new list of constraints is unique and given by:

$$d_{\text{Andre}}^* = \text{CHF}225\text{m},$$

$$d_{\text{Ben}}^* = \text{CHF}100\text{m},$$

$$d_{\text{Caroline}}^* = \text{CHF}150\text{m}$$

e) Reduce the bid amounts for all winning bidders by the discounts found in Step d); if this yields a negative bid amount, set the bid amount to zero.

105. Reducing the bid amounts of all bids made by winning bidders by their respective discounts obtained in Step d), we obtain the set of bids illustrated in Table 26.

Table 26: Modified package bids using reduced bid amounts, third iteration – Example 2

Bidder	Bid package (lots)		Bid amount
	800MHz	900MHz	
Andre	1	4	CHF275m (CHF500m – CHF225m)
	2	3	CHF250m (CHF475m – CHF225m)
Ben	0	4	CHF600m (CHF700m – CHF100m,)
	4	0	CHF250m (CHF400m – CHF150m)
Donald	0	3	CHF250m
	3	0	CHF250m
	1	4	CHF850m

f) Determine the winning bids when replacing all bids by the reduced bid amounts and calculate the corresponding sum of total winning bid amounts for this new set of winning bids.

106. We again find the combination that maximises revenues (using the reduced bid amounts), which is highlighted in orange in Table 26. The value of this allocation (using the reduced bid amounts) is CHF1.1 billion.

g) If the sum of bid amounts of the original winning bids (using the reduced bid amounts) is smaller than the sum of winning bid amounts for the new set of winning bids calculated in Step f), then winners cannot be given the discounts calculated as there exists a different combination of bidders that could provide greater revenues given the bids submitted. A new constraint is required; proceed to Step h).

Otherwise, calculate the maximum discount that can be granted to winning bidders and go to Step i).

107. The winning combination of bids found in Step f) is the same as the original winning combination of bids, and therefore the original winning bids using the reduced bid amounts yield the same value as the solution found in Step f). We can therefore calculate the maximum total discount that can be granted to winning bidders:

$$D^* = \text{CHF}225\text{m} + \text{CHF}100\text{m} + \text{CHF}150\text{m} = \text{CHF}475\text{m}.$$

108. We now proceed to Step i).

i) If there is only one combination of discounts that provide the maximum total discount given to winners, these will be used in the calculation of final base prices. Proceed to Step n).

Otherwise, we will identify a combination of discounts that distributes winner surplus evenly in relation to the maximum surplus calculated for each winner.

109. The last solution to the optimisation problem in Step d) was unique, and thus the obtained discounts can be used to calculate the final base prices. Proceed to Step n).

n) The base price for each winner is equal to the bid amount of its winning bid minus the discount.

110. We can proceed to calculate the base prices, shown in Table 27.

Table 27: Base prices for winning bids – Example 2

Bidder	Bid	Discount (Surplus)	Base price
Andre	CHF475m	CHF225m	CHF250m
Ben	CHF700m	CHF100m	CHF600m
Caroline	CHF400m	CHF150m	CHF250m

111. In this example, we must ensure that the base prices paid by winners are sufficiently large to block Donald's bid of CHF850 million for a package of one 800MHz lot and four 900MHz. Therefore:

- Andre and Ben need to pay at least CHF850 million jointly in order to outbid Donald; otherwise, the outcome would not be compliant with Rule 3.3.32⁵ as Donald expressed a willingness to pay greater than the sum of the base prices paid by Andre and Ben for a package that is a subset of the packages awarded to Andre and Ben; and
- Ben and Caroline need to pay CHF850 million jointly to outbid Donald at the same time; otherwise, the outcome would not be compliant with Rule 3.3.32⁶ given that Donald expressed a willingness to pay greater than the sum of the base prices paid by Ben and Caroline for a package that is a subset of the packages awarded to Ben and Caroline.

3.3 Example 3: Base prices selected to distribute surplus in relation to maximum surplus for each winner

112. In example 2, the maximisation of the total discount granted to winners provided a unique solution. However, this is not always the case. In this example, we find a number of solutions that provide the maximum possible total discount granted to winners. In these situations, we choose base prices so that the surplus enjoyed by winners relative to their maximum surplus is distributed evenly.
113. The setup of this example with regards to the lot structure, reserve prices and number of bidders is the same as before. In this example, bidders submit the bids shown in Table 28.

Table 28: Bids submitted – Example 3

Bidder	Bid package (lots)		Bid amount
	800MHz	900MHz	
Andre	2	3	CHF500m
Ben	1	4	CHF750m
Caroline	3	0	CHF500m
Donald	3	0	CHF400m
	0	3	CHF150m
	0	4	CHF200m

114. The value-maximising combination of winning bids is:
- Andre's bid for two 800MHz lots and three 900MHz lots (with a bid amount of CHF500 million);

⁵ See auction rules.

⁶ See auction rules.

- Ben's bid for one 800MHz lot and four 900MHz lots (with a bid amount of CHF750 million);
 - Caroline's bid for three 800MHz lots (with a bid amount of CHF500 million).
115. The combination of winning bids is highlighted in orange in Table 28. The total value of winning bid amounts is CHF1.75 billion.
116. As before, we calculate base prices to be paid by each winner following the process described in the auction rules.
- a) For each winning bidder, calculate the sum of the bid amounts that would result from the winner determination procedure if that bidder had not submitted a bid.
- b) Determine the minimum bids and the maximum discounts.
117. Maximum discount for Andre:
- *Maximum surplus:* If we discard all bids submitted by Andre, the next best allocation would include Ben's and Caroline's original winning bids, and Donald's bid for three 900MHz lots (with a bid amount of CHF150 million). The value of this combination of bids is CHF1.4 billion. Therefore, Andre's maximum surplus is CHF350 million.
 - *Maximum discount to reserve:* The reserve price for Andre's winning package is CHF106.5 million, and the maximum discount to reserve is Andre's winning bid amount (CHF500 million) minus the reserve, yielding CHF393.5 million.
 - *Maximum discount:* The maximum discount that could be granted to Andre is the minimum of CHF350 million and CHF393.5 million, which is CHF350 million.
118. Maximum discount for Ben:
- *Maximum surplus:* If we discard all bids submitted by Ben, the next best allocation would include Andre's and Caroline's original winning bids and Donald's bid for four 900MHz lots (with a bid amount of CHF200 million). The value of this combination of bids is CHF1.2 billion. Therefore, Ben's maximum surplus is CHF550 million.
 - *Maximum discount to reserve:* The reserve price for Ben's winning package is CHF106.5 million, and the maximum discount to reserve is Ben's winning bid amount (CHF750 million) minus the reserve, yielding CHF643.5 million.
 - *Maximum discount:* The maximum discount that could be granted to Ben is the minimum of CHF550 million and CHF643.5 million, which is CHF550 million.
119. Maximum discount for Caroline:
- *Maximum surplus:* If we discard all bids submitted by Caroline, the next best allocation would include Andre's and Ben's original winning bids and Donald's package bid for three 800MHz lots (with a bid amount of CHF400 million). The value of this combination of bids is CHF1.65 billion. Therefore, Caroline's maximum surplus is CHF100 million.
 - *Maximum discount to reserve:* The reserve price for Caroline's winning package is CHF63.9 million, and the maximum discount to reserve is Caroline's winning bid amount (CHF500 million) minus the reserve, yielding CHF436.1 million.
 - *Maximum discount:* The maximum discount that could be granted to Caroline is the minimum of CHF100 million and CHF436.1 million, which is CHF100 million.

Table 29: Maximum discounts and initial lower bounds for base prices – Example 3

Bidder	Maximum value of winning bids if bidder excluded	Calculation of maximum discount that can be granted to this winner	Initial lower bound for the base price to be paid by this winner
Andre	CHF1.4b	<ul style="list-style-type: none"> ▪ σ (Andre) = CHF1.75b – CHF1.4b = CHF350m ▪ $m_{\text{Andre}} = \text{CHF}106.5\text{m}$ ▪ $\rho_{\text{Andre}} = \text{CHF}500\text{m} - \text{CHF}106.5\text{m} = \text{CHF}393.5\text{m}$ $d_{\text{Andre}}^{\max} = \min(\text{CHF}350\text{m}, \text{CHF}393.5\text{m}) = \text{CHF}350\text{m}$	CHF150m (CHF500m – CHF350m)
Ben	CHF1.2b	<ul style="list-style-type: none"> ▪ σ (Ben) = CHF1.75b – CHF1.2b = CHF550m ▪ $m_{\text{Ben}} = \text{CHF}106.5\text{m}$ ▪ $\rho_{\text{Ben}} = \text{CHF}750\text{m} - \text{CHF}106.5\text{m} = \text{CHF}643.5\text{m}$ $d_{\text{Ben}}^{\max} = \min(\text{CHF}550\text{m}, \text{CHF}643.5\text{m}) = \text{CHF}550\text{m}$	CHF200m (CHF750m – CHF550m)
Caroline	CHF1.65b	<ul style="list-style-type: none"> ▪ σ (Caroline) = CHF1.75b – CHF1.65b = CHF100m ▪ $m_{\text{Caroline}} = \text{CHF}63.9\text{m}$ ▪ $\rho_{\text{Caroline}} = \text{CHF}500\text{m} - \text{CHF}63.9\text{m} = \text{CHF}436.1\text{m}$ $d_{\text{Caroline}}^{\max} = \min(\text{CHF}100\text{m}, \text{CHF}436.1\text{m}) = \text{CHF}100\text{m}$	CHF400m (CHF500m – CHF100m)

c) Generate a list of constraints and initialise it with the conditions that:

(i) for each winner, the individual discount cannot be negative (i.e., bidders cannot be asked to pay a base price that is greater than their bid amount);

(ii) for each winner, the individual discount is not greater than the maximum discount calculated in Step b); and

(iii) the aggregate discount over all winners is not greater than the reduction in the sum of bid amounts if all the bids from all winners were discarded.

120. If all bids from winning bidders were discarded, Donald would have won three 800MHz lots. The value of this allocation would be CHF400 million, and hence the maximum total discount that could potentially be granted is CHF1.35 billion (CHF1.75 billion - CHF400 million).

121. Therefore, the list of initial constraints (C) contains the following constraints:

$$d_{\text{Andre}} \geq 0,$$

$$d_{\text{Ben}} \geq 0,$$

$$d_{Caroline} \geq 0,$$

$$d_{Andre} \leq \text{CHF}350\text{m},$$

$$d_{Ben} \leq \text{CHF}550\text{m},$$

$$d_{Caroline} \leq \text{CHF}100\text{m},$$

$$d_{Andre} + d_{Ben} + d_{Caroline} \leq \text{CHF}1.35\text{b}$$

d) Maximise the total discount given to winners subject to the constraints in C.

122. The solution to this maximisation problem given the constraints is:

$$d_{Andre}^* = \text{CHF}350\text{m},$$

$$d_{Ben}^* = \text{CHF}550\text{m},$$

$$d_{Caroline}^* = \text{CHF}100\text{m}$$

e) Reduce the bid amounts for all winning bidders by the discounts found in Step d); if this yields a negative bid amount, set the bid amount to zero.

123. Reducing the bid amounts of all bids made by winning bidders by their respective discounts obtained in Step d), we obtain the set of bids illustrated in Table 30.

Table 30: Modified package bids using reduced bid amounts, first iteration – Example 3

Bidder	Bid Package (lots)		Bid amount
	800MHz	900MHz	
Andre	2	3	CHF150m (CHF500m – CHF350m)
Ben	1	4	CHF200m (CHF750m – CHF550m)
Caroline	3	0	CHF400m (CHF500m – CHF100m)
Donald	3	0	CHF400m
	0	3	CHF150m
	0	4	CHF200m

f) Determine the winning bids when replacing all bids by the reduced bid amounts and calculate the corresponding sum of total winning bid amounts for this new set of winning bids.

124. We find the combination that maximises revenues (using the reduced bid amounts), which is highlighted in orange in Table 30. The value of this allocation (using the reduced bid amounts) is CHF800 million.

g) If the sum of bid amounts of the original winning bids (using the reduced bid amounts) is smaller than the sum of winning bid amounts for the new set of winning bids calculated in Step f), then winners cannot be given the discounts calculated as there exists a different combination of bidders that could provide greater revenues given the bids submitted. A new constraint is required; proceed to Step h).

125. The sum of the reduced bid amounts of the original winning bids (shown in bold font in Table 30) is CHF750 million, which is smaller than the value of the outcome calculated in Step f). We therefore need a new constraint, so we proceed to Step h).

h) If any of the original winners is no longer among the new winning bidders calculated in Step f), then add the corresponding constraint in *C*. Go to Step d).

126. Andre and Ben, who are winners in the original outcome, are no longer among the new winning bidders calculated in Step f). To add the additional constraint to *C*, we need to calculate the joint maximum surplus of these two winners.

127. If all the bids submitted by Andre and Ben were discarded, the winning combination (using the original unmodified bids) would be:

- Caroline's bid for three 800MHz lots (with a bid amount of CHF500 million);
- Donald's bid for three 800MHz lots (with a bid amount of CHF400 million);

128. The total value of winning bid amounts in this case would be CHF900 million.

Therefore, the maximum surplus of Andre and Ben taken together is

$$\sigma(\text{Andre, Ben}) = \text{CHF}1.75\text{b} - \text{CHF}900 = \text{CHF}850\text{m}$$

and hence the maximum discount that can be granted to Andre and Ben jointly cannot exceed CHF850 million. We add the following constraint to the list of constraints:

$$d_{\text{Andre}} + d_{\text{Ben}} \leq \text{CHF}850\text{m}$$

129. We now go to Step d) and run a second iteration.

d) Maximise the total discount given to winners subject to the constraints in *C*.

130. One of the solutions to the maximisation problem given the new list of constraints is:

$$d_{\text{Andre}}^* = \text{CHF}350\text{m},$$

$$d_{\text{Ben}}^* = \text{CHF}500\text{m},$$

$$d_{\text{Caroline}}^* = \text{CHF}100\text{m}$$

131. Note that this is only one possible solution. Any combination of nonnegative discount for Andre and Ben such that $d_{\text{Andre}} + d_{\text{Ben}} = \text{CHF}850\text{m}$, $d_{\text{Andre}} \leq d_{\text{Andre}}^{\text{max}}$, $d_{\text{Ben}} \leq d_{\text{Ben}}^{\text{max}}$ and $d_{\text{Caroline}} = d_{\text{Caroline}}^{\text{max}}$ maximises the total discount given to winners. In this case, one of these solutions is picked at random.

e) Reduce the bid amounts for all winning bidders by the discounts found in Step d); if this yields a negative bid amount, set the bid amount to zero.

132. Reducing the bid amounts of all bids made by winning bidders by their respective discounts obtained in Step d), we obtain the set of bids illustrated in Table 31.

Table 31: Modified package bids using reduced bid amounts, second iteration – Example 3

Bidder	Bid package (lots)		Bid amount
	800MHz	900MHz	
Andre	2	3	CHF150m (CHF500m – CHF350m)
Ben	1	4	CHF250m (CHF750m – CHF500m)
Caroline	3	0	CHF400m (CHF500m – CHF100m)
Donald	3	0	CHF400m
	0	3	CHF150m
	0	4	CHF200m

f) Determine the winning bids when replacing all bids by the reduced bid amounts and calculate the corresponding sum of total winning bid amounts for this new set of winning bids.

133. We find the combination that maximises revenues (using the reduced bid amounts), which is highlighted in orange in Table 31. The value of this allocation (using the reduced bid amounts) is CHF800 million.

g) If the sum of bid amounts of the original winning bids (using the reduced bid amounts) is smaller than the sum of winning bid amounts for the new set of winning bids calculated in Step f), then winners cannot be given the discounts calculated as there exists a different combination of bidders that could provide greater revenues given the bids submitted. A new constraint is required; proceed to Step h).

Otherwise, calculate the maximum discount that can be granted to winning bidders and go to Step i).

134. The winning combination of bids found in Step f) is the same as the original winning combination of bids, and therefore the original winning bids using the reduced bid amounts yield the same value as the solution found in Step f). We can therefore calculate the maximum total discount that can be granted to winning bidders as

$$D^* = \text{CHF}350\text{m} + \text{CHF}500\text{m} + \text{CHF}100\text{m} = \text{CHF}950\text{m}.$$

135. We now proceed to Step i).

i) If there is only one combination of discounts that provide the maximum aggregate discount to winners, these will be used in the calculation of final base prices. Proceed to Step n).

Otherwise, we will identify a combination of discounts that distributes winner surplus evenly in relation to the maximum surplus calculated for each winner.

136. There are multiple combinations of discounts that yield the maximum total discount given to winners. Therefore, we will choose the one that minimises the differences between the winners' maximum surplus and the actual surplus that would result from applying the bid discounts.

137. To do so, we minimise the sum of squares of the difference between the maximum surplus and the discount applied for calculating base prices for each bidder (using a quadratic programming algorithm). This yields the following solution:

$$d_{Andre}^{**} = \text{CHF}325\text{m},$$

$$d_{Ben}^{**} = \text{CHF}525\text{m},$$

$$d_{Caroline}^{**} = \text{CHF}100\text{m}$$

j) Reduce the bid amounts for all winning bidders by the discounts found Step i); if this yields a negative bid amount, set the bid amount to zero.

Reducing the bid amounts of all bids made by winning bidders by the discounts obtained in Step i), we obtain the set of bids illustrated in Table 32.

Table 32: Modified package bids using reduced bid amounts, third iteration – Example 3

Bidder	Bid package (lots)		Bid amount
	800MHz	900MHz	
Andre	2	3	CHF175m (CHF500m – CHF325m)
Ben	1	4	CHF225m (CHF750m – CHF525m)
Caroline	3	0	CHF400m (CHF500m – CHF100m)
Donald	3	0	CHF400m
	0	3	CHF150m
	0	4	CHF200m

k) Determine the winning bids using the modified bid amounts and calculate the corresponding sum of winning bid amounts.

138. We find the combination that maximises revenues (using the reduced bid amounts), which is highlighted in orange in Table 32. The value of this allocation (using the reduced bid amounts) is CHF800 million.

l) If the sum of bid amounts of the original winning bids (using the reduced bid amounts) is smaller than the sum of winning bid amounts for the new set of winning bids calculated in Step k), then winners cannot be given the discounts calculated as there exists a different combination of bidders that could provide greater revenues given the bids submitted. A new constraint is required; proceed to Step m).

Otherwise, calculate the maximum discount that can be granted to winning bidders and go to Step n).

139. The winning combination of bids found in Step l) is the same as the original winning combination of bids, and therefore the original winning bids using the reduced bid amounts yield the same value as the solution found in Step k).

140. Proceed to Step n).

n) The base price for each winner is equal to the bid amount of its winning bid minus the discount.

141. We can proceed to calculate the base prices, shown in Table 33.

Table 33: Base prices for winning bids – Example 3

Bidder	Bid	Discount (Surplus)	Base price
Andre	CHF500m	CHF325m	CHF175m
Ben	CHF750m	CHF525m	CHF225m
Caroline	CHF500m	CHF100m	CHF400m

142. In this example, we must ensure that the base price paid by winners is sufficiently large as to block Donald's bid of CHF400 million for a package of three 800MHz lots. Therefore, Andre and Ben need to pay at least CHF400 million jointly in order to outbid Donald. Otherwise, the outcome would not be compliant with Rule 3.3.32⁷ as that Donald expressed a willingness to pay greater than the sum of the base prices paid by Andre and Ben for a package that is a subset of the packages awarded to Andre and Ben.

143. However, there are multiple ways of splitting the cost of outbidding Donald between Andre and Ben. In order to select a unique set of base prices, we choose the one that minimises the sum of squared differences between the maximum surplus of each bidder and the actual surplus provided by the selected base prices. This criterion always selects a unique set of base prices.

⁷ See auction rules.